

End Semester Examinations - 2015-16 Even Semester - May 2016

14MA2001 Vector Calculus and Complex Analysis

Set A

Time : 3 hrs
Total Marks: 100

1. a) Find the angle between the surfaces $x^2 - y^2 - z^2 = 11$ and $xy + yz - xz - 18 = 0$ at the point $(6, 4, 3)$. (10)
- b) Prove that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is an irrotational and find the scalar potential ϕ so that $\nabla\phi = \vec{F}$. (10)

OR

2. a) A particle moves along the curve $x = e^{-t}$, $y = \cos 3t$, $z = 2 \sin 3t$ where t is the time period. Find the component of velocity and acceleration at $t = 0$. (10)
- b) Find $f(r)$ so that $f(r)\vec{r}$ is both solenoidal and irrotational. (10)
3. a) Verify Green's theorem $\int_C (xy + y^2)dx + x^2dy$ where C is bounded by $y = x, y = x^2$. (10)
- b) Using Stoke's theorem, evaluate $\int_C (x + y)dx + (2x - z)dy + (y + z)dz$ where C is the boundary of the triangle with vertices $(2,0,0)$, $(0,3,0)$ and $(0,0,6)$. (10)

OR

4. Verify Gauss divergence theorem $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the bounded by the plane $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
5. a) Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$. (10)
- b) If $f(z)$ is an analytic, then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (10)

OR

6. a) Find the analytic function whose imaginary part is $y = -\sin x \sinh y$. (10)
- b) If $f(z)$ is analytic function with constant modulus show that $f(z)$ is constant. (10)
7. a) Find the image of the strip $1 < x < 2$ under the transformation $w = \frac{1}{z}$. (10)
- b) Find the bilinear transformation which maps the points $z = 1, i, -1$ on to the points $w = 0, 1, \infty$. (10)

OR

8. a) Find the image of the rectangle whose vertices are $A(1,1)$, $B(3,1)$, $C(3,5)$, $D(1,5)$ by the map $w=(1+i)z+3-i$.

(10)

b) Discuss the transformation $w = z + \frac{1}{z}$.

(10)

9.

a) Using Cauchy's Residue Theorem, evaluate $\oint_C \frac{z-3}{z^2+2z+5} dz$ where C is $|z+1-i|=2$.

(10)

b) Prove that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx = \frac{\pi}{3}$

(10)

Wishing you All the Best
